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Technical Note

# Mass transfer with heterogeneous chemical reaction in a Glauert flow of non-Newtonian fluid

T. Elperin [a,*], A. Fominykh [a], Z. Orenbakh [b]

[a] *The Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, P.O. Box 653, Beer Sheva 84105, Israel*
[b] *Negev Academic College of Engineering, 71 Bazel St., P.O. Box 45, Beer Sheva 84100, Israel*

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**Abstract**

Mass transfer with a heterogeneous chemical reaction of the first order in a wall jet flow for non-Newtonian power-law fluids is investigated. This problem is solved by the method of Laplace transform following the approach suggested by Apelblat [Chem. Eng. J. 19 (1980) 19]. The solution is obtained in a closed analytical form.


## 1. Introduction

Mass transfer with a heterogeneous chemical reaction of the first order occurs in chemical engineering and heterogeneous catalysis. The solution of a mass transfer problem coupled with an irreversible chemical reaction of the first order at a surface for a flow with a constant velocity was obtained in [1,2] by Apelblat. Apelblat [1] found the exact analytical solutions of a mass transfer problem with a heterogeneous chemical reaction of the first order for Couette flow and for a flow with a moving interface or a generalized Couette flow. In [2] the role of molecular diffusion in the direction of the convective transport was investigated. Diffusion with interfacial chemical reaction in a laminar channel flow was investigated by Cowherd and Haelscher [3]. Ghez [4] considered mass transport in a multicomponent system with a surface chemical reaction of the first order. In [1,5,6] a problem of mass transfer with a first-order chemical reaction at the surface between a plate and infinitive fluid flow was solved analytically by three different methods. In [7,8] the approach suggested in [1] for the solution of mass transfer problem with the first-order chemical reaction at the interface in the boundary layer flow was applied for the solution of the similar problem for Glauert flow and Falkner–Skan flow of a Newtonian fluid. The velocity components in the equation of convective diffusion used in [7,8] were adopted from [9–11]. A convective heat transfer problem with mixed boundary conditions has the same mathematical form as a convective mass transfer problem with a heterogeneous chemical reaction of the first order. In all these studies the analysis of mass transfer problems under conditions of intermediate kinetics was restricted to a case of Newtonian fluids. In the present study we considered the laminar plane jet flow of a non-Newtonian power-law fluid.

## 2. Formulation of the problem

Consider mass transfer between a solid surface and an adjacent laminar wall jet flow of a non-Newtonian power-law fluid. Semi-infinite jet emerges from a thin slot and spreads along the surface (see Fig. 1). The developed flow can be determined as a superposition of two flows: a flow over a semi-infinite plane and a jet flow

* Corresponding author. Tel.: +972-7-647-7078; fax: +972-8-647-2813.
*E-mail addresses:* elperin@menix.bgu.ac.il (T. Elperin), fominykh@bgumail.bgu.ac.il (A. Fominykh), zakhar@nace.ac.il (Z. Orenbakh).

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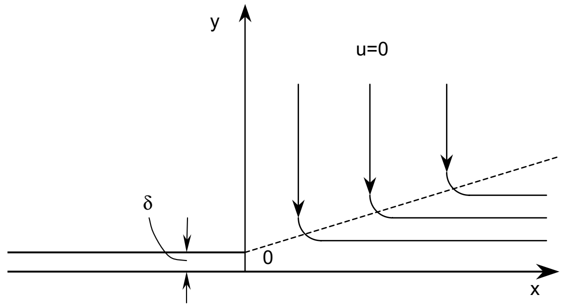


Fig. 1. Scheme of wall jet flow.

in an infinite space. The soluble material with concentration $c(x,y)$ flows with a fluid and is dispersed under the combined effects of diffusion and chemical reaction. The hydrodynamic problem for a plane wall jet flow of a power-law fluid in a semi-infinite domain of fluid was solved in [12]. The following expressions for the velocity components in $x$ and $y$ directions were derived:

$$u = B_w A_w^{(1+n)/3} x^{D_w} \cdot F'(\eta),$$
$$v = A_w^{(2n-1)/3}[k/\rho]^{-1/(1+n)} x^{E_w} \cdot \left[(1/3)\cdot D_w F + C_w \eta F'\right], \tag{1}$$

where

$$\eta = B_w A_w^{(2-n)/3}[k/\rho]^{-1/(1+n)} x^{C_w}\cdot y, \quad C_w = -\frac{3}{5n-1},$$

$$D_w = -\frac{2}{5n-1},$$

$$A_w = \left[\frac{\dot{m}}{\rho F(\infty)}\left\{\frac{B_w\delta_0}{\eta_\infty}\right\}^{\frac{D_w - C_w}{C_w}}\left(\frac{\rho}{k}\right)^{\frac{D_w}{C_w(n+1)}}\right]^{\frac{3C_w}{(n+1)C_w-(2-n)D_w}},$$

$$B_w = (5n-1)^{-1/(2n-1)}, \quad E_w = -\frac{1}{2}D_w - 1,$$

$\eta$—similarity variable, $k$—power-law consistency index, $n$—power-law behavior index in the constitutive relation for power-law fluids $\tau_{xy} = k(\partial u/\partial y)^n$, $\rho$—liquid density, $\dot{m}$—mass flow rate for the jet slot, $F(\infty)$—the value of $F(\eta)$ at the boundary layer edge where $\eta = \eta_\infty$. The unknown function $F(\eta)$ is determined from the solution of an ordinary differential equation that was derived in [13]:

$$(\mathrm{d}/\mathrm{d}\eta)\left[F''\left|F''\right|^{n-1}\right] + FF'' + 2F'^2 = 0, \tag{2}$$

where $F(0) = F'(0) = 0$, $F''(0) = 1$ and $F'(\infty) = 0$, primes denote derivatives with respect to $\eta$. For a case of large Schmidt number, $Sc = (\dot{m}/\rho\delta)^{3(n-1)/(n+1)}\cdot(k/\rho)^{2/(n+1)}/(D\cdot\delta^{(n-1)/(n+1)})$, where $\delta$ is a jet slot height, $D$—coefficient of diffusion, the thickness of the concentration boundary layer is considerably less than that of the viscous boundary layer [6]. In this case it is reasonable to assume the linear dependence of the longitudinal velocity component inside a concentration boundary layer upon $y$, i.e.,

$$u = \left(\frac{\tau(x)}{k}\right)^{\frac{1}{n}}\cdot y, \tag{3}$$

where shear stress is determined as $\tau(x) = kA_w^n B_w^{2n} a^n (k/\rho)^{-n/(1+n)} x^{n(C_w+D_w)}$, $a = F''(0)$. The transversal velocity component is found from the equation of continuity: $\partial u/\partial x + \partial v/\partial y = 0$. At steady state, neglecting diffusion in the direction of the convective transport, the mass transfer is governed by the following convective–diffusion equation:

$$\left(\frac{\tau(x)}{k}\right)^{\frac{1}{n}}\cdot y\frac{\partial c}{\partial x} - \frac{y^2}{2n\cdot k^{1/n}}[\tau(x)]^{(1-n)/n}\frac{\partial\tau(x)}{\partial x}\cdot\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}, \tag{4}$$

with the following initial and boundary conditions

$$c = c_0 \quad \text{for } x = 0 \text{ and } y > 0, \tag{5}$$

$$c = c_0 \quad \text{for } x > 0 \text{ and } y \to \infty, \tag{6}$$

$$D\frac{\partial c}{\partial y} = k_s c \quad \text{for } x > 0 \text{ and } y = 0, \tag{7}$$

where $c(x,y)$ is a molar concentration at $x$, $y$, $c_0$—concentration in the bulk of liquid, $k_s$—rate constant of a surface chemical reaction.

## 3. Solution

To solve the boundary value problem (4)–(7) let us introduce new variables, related to $x$ and $y$ by the following formulas (see [6]):

$$Y = 2^{-1/3}\left(\frac{\tau(x)}{k}\right)^{\frac{1}{2n}} y, \quad X = \frac{2\alpha}{C_w + D_w + 2} x^{\frac{C_w+D_w+2}{2}}, \tag{8}$$

where $X$ and $Y$ are transformed coordinates in the direction parallel and normal to the interface, $\alpha = 2^{1/2}A_w^{1/2}B_w a^{1/2} D(k/\rho)^{-1/2(1+n)}$. Then Eq. (4) reduces to Leveque equation

$$Y\frac{\partial C}{\partial X} = \frac{\partial^2 C}{\partial Y^2} \tag{9}$$

where $C = c - c_0$. The boundary conditions (5)–(7) in the new variables read:

$$C = 0 \quad \text{for } X = 0 \text{ and } Y = 0, \tag{10}$$

$$C = 0 \quad \text{for } X > 0 \text{ and } Y \to \infty, \tag{11}$$

$$\frac{\partial C}{\partial Y} = \beta X^{-\frac{C_w+D_w}{C_w+D_w+2}} C + \beta X^{-\frac{C_w+D_w}{C_w+D_w+2}} c_0 \quad \text{for } Y = 0, \tag{12}$$

where

$$\beta = 2^{1/3} k_s 2^{(C_w+D_w)/(C_w+D_w+2)} \cdot (C_w + D_w + 2)^{-(C_w+D_w)/(C_w+D_w+2)} \cdot \alpha^{-2/(C_w+D_w+2)}.$$

Applying the Laplace transform to Eq. (9), using the boundary conditions (10), (11) and following [1], one obtains:

$$C(X,Y) = \int_0^X G(t) \int_0^\infty \frac{Y^{1/2} u^{1/6}}{2\sqrt{3}\pi} \times \exp[-u(X-t)] J_{1/3}(\phi)\, du\, dt, \tag{13}$$

where $\phi = 2u^{1/2} \cdot Y^{3/2}/3$, $J_{1/3}(\phi)$—Bessel function of the first kind. The unknown function $G(X)$ is determined from the remaining boundary condition (12). Eqs. (12) and (13) yield the following integral Abel equation for evaluating $G(X)$:

$$\int_0^X \frac{G(t)dt}{(X-t)^{4/3}} = \gamma X^{-\frac{C_w+D_w}{C_w+D_w+2}}, \tag{14}$$

where $\gamma = 3^{5/6} \cdot 2\pi c_0 \beta$, $\Gamma(z)$—gamma function. The solution of this Abel equation has the following form (for details see [14], Chapter 6):

$$G(X) = -\frac{(C_w + D_w)}{(C_w + D_w + 2)} \frac{\gamma 3^{\frac{1}{2}}}{2\pi} \times \int_0^X t^{1/3} (X-t)^{-\frac{2C_w+2D_w+2}{C_w+D_w+2}} dt. \tag{15}$$

Eq. (15) is valid only for $n > 0.7$. Expressing the integral in Eq. (15) through the gamma function (see, e.g., [15])

$$\lambda_3 = \frac{3^{-\frac{1}{2}} 2^{\frac{2}{C_w+D_w+2}} \pi^{\frac{1}{2}} \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{4-C_w-D_w}{3(C_w+D_w+2)}\right) \Gamma\left(-\frac{C_w+D_w}{C_w+D_w+2}\right) \cdot \left(\frac{B_w}{\eta_\infty}\right)^{\frac{1}{3}} F(\infty)^{\frac{3}{5n-1}}}{(C_w+D_w+2)^{\frac{3}{2}} \cdot \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{C_w+D_w+8}{3(C_w+D_w+2)}\right) B_w^{\frac{2}{3}} a^{\frac{1}{3}}}. \tag{18}$$

and calculating the integrals in Eq. (13) we arrive at the analytical expression for the distribution of concentration in the wall jet flow with the first order surface chemical reaction:

$$\frac{c(X,Y)}{c_0} = 1 - \frac{3^{\frac{5}{6}} \lambda_1 X^{7/3}}{Y} \Gamma\left(\frac{4-C_w-D_w}{3(C_w+D_w+2)}\right) X^{\frac{4-C_w-D_w}{3(C_w+D_w+2)}} \times \exp\left(-\frac{Y^3}{18X}\right) W_{\frac{C_w+D_w-4}{3(C_w+D_w+2)}, -\frac{1}{6}}\left(-\frac{Y^3}{9X}\right), \tag{16}$$

where

$$\lambda_1 = \frac{\gamma(C_w+D_w)\Gamma(1/3)\Gamma(-(C_w+D_w)/(C_w+D_w+2))}{6 \cdot \pi^{3/2} \cdot (C_w+D_w+2) \cdot \Gamma((C_w+D_w+8)/(3(C_w+D_w+2)))},$$

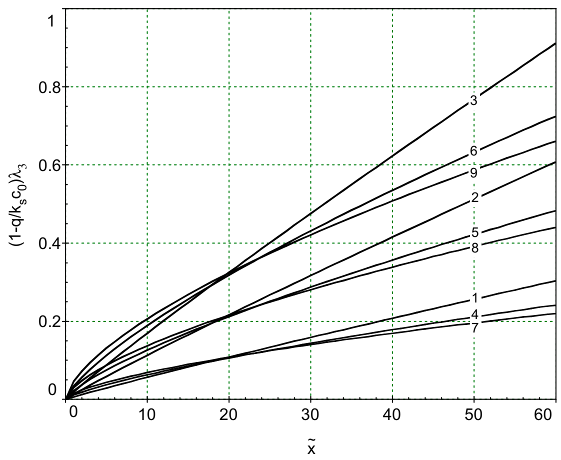


Fig. 2. Dependence of the dimensionless mass flux $(1 - q/k_s c_0)/\lambda_3$ vs. dimensionless coordinate $\tilde{x}$ for $Re = 10$, $Sc = 500$, Eq. (17): (1) $n = 0.75$, $Da = 0.5$; (2) $n = 0.75$, $Da = 1.0$; (3) $n = 0.75$, $Da = 1.5$; (4) $n = 1.0$, $Da = 0.5$; (5) $n = 1.0$, $Da = 1.0$; (6) $n = 1.0$, $Da = 1.5$; (7) $n = 1.75$, $Da = 0.5$; (8) $n = 1.75$, $Da = 1.0$; (9) $n = 1.75$, $Da = 1.5$.

$W_{\ell,m}(z)$—Whittaker function. From Eq. (16) we obtain the explicit formula for the mass flux density at a wall:

$$\frac{q}{k_s} c_0 = 1 - \lambda_3 \frac{Da}{Re^{\frac{(5n+1)}{(1+n)(5n-1)}}} \frac{\tilde{x}^{\frac{4+5n}{3(5n-1)}}}{Sc^{\frac{1}{3}}}, \tag{17}$$

where $Da = k_s\delta/D$—Damkohler number, $Re = (\dot{m}/\rho\delta)^{2-n}\delta^n(k/\rho)^{-1}$—Reynolds number, $\tilde{x} = x/\delta$,

Eq. (17) implies that at a location where a jet emerges from a slot, the mass flux is determined only by a rate of a chemical reaction. Convective mass transfer in the liquid acquires a more significant role as the distance from a slot increases. The dependence of dimensionless mass flux on the dimensionless coordinate $\tilde{x}$ given by Eq. (17) is shown in Fig. 2 for $Sc = 500$, $Re = 10$, $Da = 0.5$, 1.0 and 1.5 for values of rheological parameter $n$ equal to 0.75, 1.0 and 1.75.

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